Question (1973 STEP II Q15)

A particle of unit mass falls from a position of unstable equilibrium at the top of a rough sphere of radius a. Show that the equations of motion may be written $a\omega\frac{d\omega}{d\theta} = g\sin\theta - \mu R$, $R = g\cos\theta - a\omega^2$, where θ is the inclination to the upward vertical of the line from the particle to the centre of the sphere, $\omega = \dot{\theta}$, and R is the reaction of the sphere on the particle. Show that if $\mu = 0$, the particle leaves the sphere at $\theta = \alpha$, where $\cos\alpha = \frac{2}{3}$. Now suppose μ is positive but small. Solve the first equation approximately by giving $R(\theta)$ the value it has in the solution for $\mu = 0$. Hence obtain an improved formula for $R(\theta)$, and by regarding the required value of θ as α plus a small correction, show that the particle leaves the sphere where $\theta = \alpha + \mu\left(2 - \frac{4\alpha}{3\sin\alpha}\right)$ approximately. [Use the facts that, if x is small, $\sin x$ and $\cos x$ can be approximated by x and 1, respectively.]

Question (1972 STEP III Q10)

The ends of a uniform rod of length $2b$ are constrained to lie on a smooth wire in the form of a parabola, which is fixed with its axis vertical and vertex downwards. The length of the latus rectum of the parabola is $4a$. Prove that, when the rod is inclined at an angle θ to the horizontal, the height of its centre of gravity above the vertex of the parabola is

$$\frac{1}{4a}(b^2\cos^2\theta + 4a^2\tan^2\theta).$$

Hence, or otherwise, find the positions of equilibrium and discuss their stability.

Question (1969 STEP IIII Q16)

Three unequal rods A_0A_1, A_1A_2 and A_2A_3 are smoothly jointed at A_1 and A_2. The ends A_0 and A_3 can slide along a smooth horizontal rail. Find the position of stable equilibrium. Investigate the equilibrium of a similar system consisting of a chain of n rods $A_0A_1, \dots, A_{n-1}A_n$, and show that there is precisely one stable configuration.

Question (1972 STEP IIII Q13)

A heavy, uniform circular cylinder of radius r lies on a rough horizontal plane with its axis horizontal. A heavy, uniform rod AB of length l lies in the vertical plane which bisects the axis of the cylinder at right angles. Its end A rests on the plane and point C, distinct from B, is in contact with the cylinder. The coefficient of friction is the same value, μ, at each of the three points of contact between the rod, cylinder and plane. The rod makes an angle α with the horizontal, and the friction is limiting at Show, by a geometrical method or otherwise, that for fixed values of r, l and α, there exists a value of μ such that this situation is a possible equilibrium state of the system provided

$$r\cot(\frac{1}{2}\alpha) \leq l\cos\alpha \leq 2r\cot(\frac{1}{2}\alpha).$$

Question (1974 STEP IIII Q14)

A rectangular sheet of adhesive material is placed with its adhesive side uppermost on a plane which is inclined to the horizontal at an angle α. Two opposite edges of the sheet (of length b) are horizontal. The coefficient of friction between the sheet and the plane is sufficient to prevent slipping. The material has uniform mass per unit area m, and its thickness is negligible. A uniform solid cylinder of mass M, radius a and length b is placed along the top edge of the material and released from rest. The material then adheres to the cylinder as it rolls downwards. Show that when the cylinder has rolled through an angle θ, and an area $ab\theta$ of material has been lifted from the plane, the potential energy of the system has decreased by an amount

$$Mga\theta \sin\alpha - mga^2b[(\theta - \sin\theta)\cos\alpha + (1 - \frac{1}{2}\theta^2 - \cos\theta)\sin\alpha].$$

By examining the approximate form of this expression when θ is small, find an approximation to a value of θ for which equilibrium is possible when $M\tan\alpha$ is small compared with mab.

Question (1962 STEP IIII Q110)

A light strut of length a is freely pivoted at one end A, and the other end B carries a light small pulley. A light rope passes over the pulley; one end is fixed at a height h vertically above A, and the other end carries a weight. Discuss the positions of equilibrium and their stability.

Question (1961 STEP II Q207)

Describe briefly the laws of friction as applied to simple problems in mechanics. A particle of mass m_1 is attached to another of mass m_2 by a smooth, light, rigid rod. The system is placed on a rough inclined plane carrying a smooth pin which passes through the hole in the centre of the rod. Show that equilibrium is possible with the rod at any angle in the plane if

$$\mu > \frac{|m_1 - m_2|}{m_1 + m_2}\tan\alpha,$$

where μ is the coefficient of friction between the particles and the plane, and α is the inclination of the plane to the horizontal. Write down the equation of motion for the case when the rod is made to rotate in the inclined plane by the application of a couple G.

Question (1962 STEP II Q207)

A uniform rod of length $2a$ is smoothly hinged at one end to a fixed point A of a horizontal axis in such a way that it must lie in the plane through A perpendicular to the axis. The rod is kept in rotation with constant angular velocity ω about the vertical through A. Show that, if ω is sufficiently large, there is a position of relative equilibrium in which the rod makes an angle α with the vertical, given by $\cos\alpha = 3g/(4\omega^2)$. Show also that the equilibrium is stable, and that the period of small oscillations about the equilibrium position is $2\pi/(\omega\sin\alpha)$. Derive corresponding results for the case in which the rod is non-uniform.

Question (1958 STEP II Q307)

A uniform solid hemisphere is balanced in equilibrium with its curved surface in contact with a sufficiently rough inclined plane. Find the greatest possible value of the inclination of the plane to the horizontal, and less inclination there may be two positions of equilibrium, one stable and one unstable. Show also that the coefficient of friction has to be greater than $3/\sqrt{(55)}$ for the plane to be sufficiently rough.

Question (1959 STEP II Q308)

A particle moves in a circle of radius a about a centre of force which exerts an attraction of magnitude μr^n, r being the distance from the centre. By considering first-order equations for the time variation of the quantity ϵ, where $r = a(1+\epsilon)$ and ϵ is considered small, discuss the stability of this motion when it is disturbed

(a) by a small transverse impulse,

(b) by a small radial impulse.

Question (1959 STEP III Q104)

A rigid hoop, of radius a, is made of thin smooth wire, and is fixed with its plane vertical. A small bead, of weight w, is free to slide on the hoop and is joined to one end of a light elastic string, of modulus of elasticity λ. The other end of the string is fastened to the highest point of the wire, and the unstretched length l of the string is less than $2a$. Show that asymmetrical positions of equilibrium exist if $2lw/\lambda < 2a - l$. If these asymmetrical positions exist, show that the equilibrium is stable.

Question (1959 STEP III Q108)

Two small smooth pegs are situated at a distance $2h$ apart at the same level. A light string, which hangs symmetrically across the pegs, carries a mass m at each end and a mass M $(< 2m)$ at the middle point. If the masses move vertically and the string remains taut and below the level of the pegs, write down the kinetic energy and the potential energy in terms of the angle θ that the inclined portions of the string make with the vertical. By writing $\theta = \alpha + \psi$, where α is the equilibrium value of θ and ψ and its derivatives with respect to the time are assumed to remain small, deduce from the energy equation that the period of small vertical oscillations of the system is the same as that of small oscillations of a simple pendulum of length $h \cos\alpha \cot\alpha(\cos\alpha + \cot\alpha)$.

Question (1961 STEP III Q110)

A rigid plank of length l, breadth b and thickness h is laid across a rough log of radius r to act as a seesaw. Find the relationship between r and h for the plank to rest stably on the log if it is initially placed symmetrically across it. Could you have solved this problem by any other methods? If so, describe them briefly.

Question (1964 STEP III Q109)

A fixed hollow sphere of radius a has a small hole bored through its highest point, resting on the inside of the sphere; there is no friction. Find, for any value of b/a, how many positions of equilibrium there are with the rod in a given vertical plane and which of them are stable.

Question (1960 STEP III Q209)

A uniform solid consists of a hemisphere of radius a to the base of which is fixed, symmetrically, a circular cylinder of radius a and length $\frac{1}{4}a$. The solid rests in equilibrium with its axis vertical, on a rough sphere of radius b. Show that, if the cylindrical part of the solid is uppermost, the equilibrium is stable if, and only if,

$$b > \frac{4}{5}a.$$

Question (1962 STEP III Q202)

A uniform heavy rod AB of length $2l$ can turn freely about a fixed point A, and C is a fixed point at height $2a$ vertically above A. A small heavy ring of weight equal to that of the rod can slide smoothly along the rod, and is attached to C by a light inelastic string of length a. Given that $a < 2l$, prove that the configuration in which the rod is vertical is one of stable equilibrium if $a > l$.

Question (1963 STEP III Q201)

A uniform rectangular rough plank of weight W and thickness $2b$ rests in equilibrium across the top of a fixed horizontal circular cylinder of radius a. The length of the plank is perpendicular to the axis of the cylinder. Find an expression for the increase in potential energy if the plank is turned without slipping through an angle θ, and deduce that the horizontal position is stable provided that $a > b$. Discuss the case $a = b$.

Question (1958 STEP III Q305)

A smooth wire has the shape of a parabola whose latus rectum is of length l_0 and whose axis is vertical and vertex upwards. Two beads A and B, whose masses are m_1 and m_2, where $0 < m_1 < m_2$, slide on the wire, and are joined by a light inelastic string of length l, where $l > 2a$, which passes through a small smooth ring at the focus of the parabola. Prove that the only positions of equilibrium for which the two beads are not on the same side of the vertex are those in which at least one of the beads is at the vertex of the parabola, and determine which positions are stable and which are unstable. How is the problem altered if $m_1 = m_2$?

Question (1959 STEP III Q306)

A bead of mass m is free to move on a smooth circular wire of radius r which is fixed in a vertical plane. A light, perfectly elastic string of natural length r has one end attached to the bead and the other fixed to the highest point P of the wire. The tension in the string when the bead is at the lowest point Q of the wire is T_0. Show that there are positions of equilibrium other than P and Q if and only if $T_0 > 2mg$, and that in this case equilibrium at Q is unstable. In the particular case $T_0 = 4mg$ the bead is slightly disturbed from rest at Q. Show that in the subsequent motion the string becomes slack, and find the reaction of the wire on the bead at the instant when this occurs.

None

Question (1960 STEP III Q302)

A plane framework consists of five uniform heavy rods AB, BC, CD, DA, AO, smoothly hinged at A, B, C and D, and two light elastic strings BO and DO; O lies outside the rhombus $ABCD$. Each rod is of length l and weight W, and each string has natural length l and modulus $6W$. The framework is suspended freely from O. Show that there is a position of equilibrium in addition to that in which the rods are all vertical, and examine the stability of these two positions of equilibrium. Find the reaction at C between the rods BC and CD when the framework hangs in the equilibrium position in which the rods are not all vertical.

Question (1961 STEP III Q301)

A uniform thin rod of length $2a$ and weight W is freely hinged at one end to a fixed support. The other end is joined by a light elastic string of modulus λ and unstretched length l ($l < 4a$) to a fixed hook located vertically above and at distance $2a$ from the hinge. Find the range of values of λ for which there exists a position of equilibrium with the rod inclined to the vertical. Show also that if such a position of equilibrium exists then it is stable.

Question (1962 STEP III Q302)

A stiff rod AB of length a pivots about a fixed point A and is attached by an elastic string, of unstretched length a and modulus E, to a fixed point C at a distance a from A. A force parallel to and in the direction of CA, of magnitude equal to $E/8a$ times the length BC, is applied to the rod at B, the rod being constrained to lie in a fixed plane through AC. Show that there exists a position of equilibrium where the rod is not parallel to the string. Show also that this position is stable.

Question (1963 STEP III Q302)

A uniform rod of length l_0 and mass m is hinged at one end to the point A and is free to rotate in a vertical plane through the point B which is at a distance l_0 horizontally from A. The other end of the rod is attached to the point B by an elastic string of unstretched length l_0. The tension in the string when stretched to a length $l(l > l_0)$ is given by $mgR(l - l_0)/l_0$. Derive an equation in terms of θ, the angle between the rod and the line AB, for the positions of equilibrium of the system with the rod lying below AB, and obtain a criterion for their stability. If $f'(\omega) > 0$ for all $\omega > 0$, show that only one such position exists, and discuss its stability.

Question (1959 STEP III Q402)

A uniform heavy rod of length $2b$ has its ends attached to small light rings which slide on a smooth rigid wire in the shape of a parabola of latus rectum $4a$ held fixed in a vertical plane with its vertex uppermost. Prove that the horizontal position of the rod is one of stable equilibrium if $b > 2a$. Show further that in this case there are two oblique positions of equilibrium, one on either side.

Question (1961 STEP III Q404)

A bead of mass m, which is free to move on a smooth wire in the form of an ellipse held fixed in a vertical plane with its major axis vertical, is attached to one end of a light elastic string of modulus λ whose other end is attached to the uppermost focus of the ellipse, so that when the particle is at the top end of the major axis of the ellipse the string is just taut. Find the possible positions of equilibrium, and show that there is an oblique position if

$$\frac{\lambda}{mg} > \frac{1-e}{2e^2},$$

where e is the eccentricity of the ellipse.

Question (1965 STEP III Q2)

A uniform rod of length l has a ring at one end which slides on a smooth vertical wire. A smooth cylinder of radius a has its axis horizontal and at a distance b ($b > a$) from the wire. The rod rests in tangential contact with the cylinder in a plane perpendicular to its generators. Find a relation between l and the angle θ which the rod makes with the wire. Deduce that if $l > 2b$ there exist two positions of equilibrium for all values of a.

Question (1966 STEP III Q3)

A uniform solid parabolic cylinder, whose cross-section consists of the area in the (x, y) plane defined by the inequalities

$$y^2 \leq 4ax,$$

$$x \leq h,$$

where a and h are positive constants, rests with its curved surface in contact with a rough horizontal plane. Show that if $h < 10a/3$ it can rest in stable equilibrium with its plane surface horizontal. What are the possible positions of equilibrium when $h > 10a/3$? State (with reasons) which of them are stable.

Question (1950 STEP II Q207)

A uniform beam of thickness $2c$ rests horizontally upon a fixed perfectly rough circular cylinder of radius a whose axis is horizontal and perpendicular to the direction of the length of the beam. The beam is then rolled on the cylinder keeping its length perpendicular to the axis of the cylinder. Show that there is a position of equilibrium with the beam inclined to the horizontal if $a > c$ and that this position is unstable.

Question (1951 STEP II Q207)

A long narrow hollow tube is inclined at an angle α to the vertical, and a particle of mass m which lies inside the tube is attached to a point O of the tube by an elastic string of modulus λ and natural length a. The vertical plane in which the tube lies is rotated about a vertical axis through O with constant angular velocity ω. Assuming that the particle lies below the point O, find the least value λ_0 of λ which will ensure that the string does not stretch indefinitely. Show that for $\lambda > \lambda_0$ the particle can oscillate about its position of equilibrium with a period $2\pi\sqrt{[am/(\lambda - \lambda_0)]}$.

Question (1955 STEP II Q207)

A uniform plank of thickness $2h$ is placed on top of a perfectly rough fixed circular cylinder of radius a $(h < a)$. The axis of the cylinder is horizontal and the long edges of the plank are perpendicular to the axis of the cylinder. When the plank is rolled into the horizontal position its centre of gravity is a distance l from the vertical plane through the axis of the cylinder. Prove that there are three positions of equilibrium in which the plank rests on the cylinder, provided that $l < l'$, where

$$l' = a \arccos(h/a)^{\frac{1}{2}} - ((a-h)h)^{\frac{1}{2}}$$

and show that only the intermediate position is stable. Examine the case $l > l'$.

None

Question (1956 STEP II Q209)

Find in terms of polar coordinates (r, θ) the radial and transverse velocities and accelerations of a particle moving in a plane. A particle is attracted to the fixed point O by a force $\mu f(u)$ per unit mass, where $u = r^{-1}$. Prove that the path of the particle satisfies the equation

$$\frac{d^2u}{d\theta^2} + u = \frac{\mu f(u)}{h^2u^2},$$

where h is a constant. Find the condition that the particle may move in a circle $u = c$, and by taking $u = c + \xi$, where ξ and its derivatives are small, show that the circular motion is stable for small disturbances provided that $cf'(c) < 3f(c)$.

Question (1957 STEP II Q207)

Two equal circular cylinders of radius r lie fixed with their axes parallel at distance d apart in a horizontal plane. A smooth prism, whose right section is a square of side $2l$, is placed with adjacent faces resting on the two cylinders, its axis being parallel to those of the cylinders. Find, in terms of l and r, the greatest value of d for which no unsymmetrical configuration of equilibrium can exist. Show that when an unsymmetrical configuration exists it is unstable. Examine the stability of the prism in its symmetrical position.

Question (1954 STEP II Q310)

The pendulum of a clock consists of a uniform rod AB, of length $2a$ and mass M, freely suspended from the end A, and a particle of mass $\frac{1}{3}M$ is attached to the rod at a distance $a(1+x)$ from A. Prove that the period of a small oscillation about the position of stable equilibrium is

$$2\pi\sqrt{\left(\frac{5+2x+x^2}{4+x}\right)\frac{a}{g}}.$$

Prove that, if x is small, the period is approximately

$$\frac{2\pi}{n}\left(1+\frac{3}{40}x^2\right),$$

where $n^2 = 4g/(5a)$. If the clock keeps good time when $x = 0$, how many seconds will it lose in a day when $x = 1/1000$?

Question (1956 STEP II Q306)

Explain the principle of virtual work and discuss its application to problems in statics. Twelve equal uniform rods of weight w and length l are freely jointed to form the edges of a regular octahedron. The system is suspended from one corner and a weight W is attached to the opposite corner. Show that the thrust in a horizontal rod is

$$(W+6w)/2\sqrt{2}.$$

Question (1951 STEP III Q103)

A rough circular cylinder of radius r is fixed with its axis horizontal. A uniform cubical block of weight W with edges of length a is placed symmetrically upon the cylinder with four edges vertical and four edges parallel to the axis of the cylinder. The block is then rolled upon the cylinder, without slipping, until it has turned through an angle θ. Calculate (i) the work V done against gravity in this rotation, (ii) the moment M of the couple needed to hold the block in equilibrium in the position θ, indicating the sense of this moment. Hence determine the condition that the position of equilibrium $\theta = 0$ should be stable.

Question (1952 STEP III Q104)

A circular cylinder of radius a is fixed with its axis horizontal. On the cylinder rests a thick plank with its length horizontal and perpendicular to the generators and its centre of gravity at a height b vertically above the highest generator. The surfaces are sufficiently rough to prevent slipping. Show that the plank is in stable or unstable equilibrium according as $b < a$ or $b \geq a$.

None

Question (1953 STEP III Q101)

Three unequal uniform rods AB, BC and CD, of lengths a, b and c respectively, are smoothly jointed together at B and C; the ends A and D slide smoothly along a horizontal rail. Find the positions of equilibrium, and illustrate by sketches the various cases.

Question (1954 STEP III Q104)

Two equal straight light rods AB, BC, each of length l, are freely hinged together at B, where they carry a mass M; masses m are borne at A and at C. The rods rest symmetrically across two smooth parallel horizontal rails, of small cross-section, fixed at a distance $2c$ apart and at the same level. Show that a position of equilibrium exists in which the plane of the rods is perpendicular to the rails and B is at a higher level than A and C, provided that M is less than $2m(l-c)/c$. Show that if this position of equilibrium exists it is stable for displacements in which the plane of the rods remains perpendicular to the rails and B is displaced vertically. Investigate the corresponding problem with B below AC.

Question (1957 STEP III Q104)

A uniform plank of thickness t is placed symmetrically across a rough fixed horizontal log whose cross-section is a circle of radius a, and rests in equilibrium in a horizontal position. Discuss its stability for all values of t/a.

Question (1950 STEP III Q202)

A uniform heavy straight tube is supported by an endless light string which passes through the tube (in which it fits loosely) and over a fixed smooth peg. The inside of the tube is rough except that the ends of the tube are perfectly smooth. Find the positions of equilibrium. Also find the positions of equilibrium if the string is cut and the two ends are fastened to the peg.

Question (1953 STEP III Q202)

Two thin rods AB, BC are fixed together at B, the angle ABC being $105°$. The rods are in a vertical plane with B below A and C, the rod AB being inclined to the horizontal at $45°$. A uniform thin rod XY of mass M is in equilibrium with its ends X, Y attached to AB and BC respectively by light smooth rings. Determine whether the equilibrium is stable or unstable.

Question (1954 STEP III Q204)

A particle of weight W is free to move on a smooth elliptical wire fixed with its major axis, of length $2a$, vertical and its minor axis, of length $2a/\sqrt{3}$, horizontal. The particle is attracted towards the highest point of the ellipse by a force which is of magnitude Wr/a when the particle is distant r from the highest point. Obtain an expression for the potential energy of the particle when its coordinates, referred to vertical and horizontal axes through the centre of the ellipse, the positive direction of the vertical axis being downwards, are

$$\left(a\cos\theta, \frac{a}{\sqrt{3}}\sin\theta\right).$$

Hence, or otherwise, show that the values $\theta = 0$ and $\theta = \pi/3$ give positions of equilibrium, and determine the stability of each of them.

Question (1957 STEP III Q202)

A uniform solid elliptic cylinder is in stable equilibrium resting on a perfectly rough horizontal table. Show that no amount of loading along its highest generator will render it unstable if the eccentricity of its cross-section exceeds a certain value, to be found.

Question (1951 STEP III Q305)

A bead of weight w is threaded on a smooth circular wire of radius a which is fixed in a vertical plane. A light elastic string of unstretched length a and modulus of elasticity λ joins the bead to the highest point of the wire. If $\lambda > 2w$, prove that there are three positions of equilibrium of the bead on the wire (with the string stretched), and discuss the stability of each position.

Question (1950 STEP III Q404)

A uniform perfectly rough heavy plank of thickness $2b$ rests symmetrically across the top of a fixed horizontal circular cylinder of radius a, the length of the plank being perpendicular to the axis of the cylinder. Show that if the plank is turned without slipping through an angle θ round the direction of the axis of the cylinder, the gain of potential energy is proportional to

$$a\theta\sin\theta - (a+b)(1-\cos\theta),$$

and find the condition that the horizontal position shall be stable. Show further that if $a > b$ there always exists a second position of equilibrium into which the plank can be turned.

Question (1953 STEP III Q402)

A non-uniform sphere of radius a rests in equilibrium on top of a fixed sphere of radius b. The surfaces are perfectly rough and the centre of mass of the first sphere is at a distance d above the point of contact. Find the vertical distance through which the centre of mass rises when the first sphere is made to roll in contact with the other until the radius to the original point of contact is inclined at an angle θ to the vertical. Hence, or otherwise, prove that the equilibrium is stable if

$$\frac{1}{d} > \frac{1}{a} + \frac{1}{b}.$$

Question (1954 STEP III Q402)

A cylinder of radius a is such that its centre of gravity G is at distance r from its axis. The cylinder can roll on a perfectly rough plane of inclination α with its axis horizontal. Show that if $r > a \sin \alpha$, equilibrium is possible in two different positions, and that in each of them the plane through G and the axis of the cylinder makes the same angle with the vertical. Show that one of these positions corresponds to stable equilibrium and one to unstable equilibrium.

Question (1955 STEP III Q402)

A smooth uniform heavy sphere of weight W and radius a suspended from a point O by a light string of length l whose other end is attached to a point A of the spherical surface rests in stable equilibrium in contact with a smooth plane through O and inclined at an angle α to the vertical. Prove that the tension T in the string is given by

$$T = W(l + a) \cos \alpha / [l(l + 2a)]^{\frac{1}{2}}.$$

Prove that if the sphere (still of uniform material) were hollow with an internal smooth concentric spherical cavity containing smooth weights free to move, the position of stable equilibrium is the same as before, and determine the tension in the string in terms of the weights of the bodies involved.

Question (1957 STEP III Q404)

A non-uniform rigid rod has its ends attached to light rings which can slide on a rigid rough wire in the shape of a circle held in a vertical plane. The coefficient of friction at both ends is μ and is less than unity, and the rod subtends a right angle at the centre of the circle. Find the range of values for the angle of inclination to the horizontal of the rod in equilibrium for varying positions of its centre of mass, and verify that the range of values varies between 2λ and 4λ, where $\tan \lambda = \mu$.

Question (1947 STEP II Q207)

Four uniform rods each of length l and weight W are smoothly jointed to form a rhombus $ABCD$. The ends of a light elastic string of unstretched length l are tied to the joints A and C and the system hangs in equilibrium from A. The string is such that the tension is $4W(x-l)/l$ when the length x exceeds l. Show that there is a position of equilibrium in which each rod makes the same angle with the vertical. What is this angle?

Question (1947 STEP III Q109)

Prove that the centre of gravity of a uniform solid hemisphere of radius a is at distance $\frac{3}{8}a$ from the plane face.
A solid hemisphere of radius a rests symmetrically upon a fixed sphere of radius b; the plane face of the hemisphere is horizontal and uppermost, and the centre of this face is vertically above the centre of the sphere. Discuss the stability of the position of equilibrium for all possible values of a/b, assuming that no slipping occurs.

Question (1947 STEP III Q305)

A smooth bead of mass m is free to slide on a circular wire of radius a, which is fixed in a vertical plane. The bead is joined to the highest point of the circle by a light elastic string of modulus λ and natural length ka (where $k < 2$). Show that there are three positions or one position of equilibrium with the string taut, according as

$$k < \frac{2\lambda}{\lambda + 2mg} \quad \text{or} \quad k \geq \frac{2\lambda}{\lambda + 2mg}.$$

Investigate the stability of equilibrium in each case.

Question (1923 STEP II Q813)

Prove that

$$\left(\frac{dp}{dT}\right)_v = \frac{l_v}{T},$$

where p, v, T are respectively pressure, volume and temperature, and l_v is the latent heat of expansion at temperature T. Defining a perfect gas as one in which pv and the internal energy are both functions of T only, show that the coefficient of cubical expansion at constant pressure is the same for all perfect gases at the same temperature. Show further that the difference of the specific heats is for the same perfect gas a constant.