Question (1976 STEP II Q7)

Balls are drawn successively at random without replacement from a box containing R red balls and B blue ones. Find the probability that the number of balls to be drawn in order to obtain r red ones $(r \leq R)$ should be n.

Question (1975 STEP III Q9)

A man tosses a coin until he tosses a head for the nth time. The number of tosses he makes is denoted by N. Show that the probability that $N < 2n - 1$ is $\frac{1}{2}$, and find the expected value of N.

Question (1963 STEP IIII Q104)

If a fair coin (i.e. one without bias) is tossed n times, show that the probability that r heads and $(n - r)$ tails occur is

$$\frac{n!}{r!(n-r)!} 2^{-n}.$$

An experimenter decides to continue tossing a fair coin until k heads have occurred. Find the probability p_n that he will have to perform exactly n tosses, and show that p_n is the coefficient of z^n in the power series expansion of

$$z^k (2 - z)^{-k}.$$

Deduce that

$$\sum_{n=k}^{\infty} p_n = 1$$

and interpret this result.