Question (1916 STEP I Q103)

Write a short essay on complex numbers, starting from the beginning and erecting a series of definitions and theorems sufficient to justify the manipulation of complex numbers in accordance with all the laws of elementary algebra.

None

Question (1918 STEP I Q701)

Complex Numbers.

Question (1918 STEP I Q702)

The Exponential and Logarithmic Functions of a real variable.

Question (1918 STEP I Q703)

Starting from the existence of real numbers, and Dedekind's theorem concerning sections of real numbers, state, without proof, the chain of theorems leading to the proposition that any continuous function is integrable. Establish the principal properties of integrals. Show that if $f(x) = 0$ for irrational values of x, and $f(x) = 1/q$ when x is a rational p/q, where p/q is in its lowest terms, then $f(x)$ has in any finite interval a Riemann integral, whose value is zero.

Question (1918 STEP I Q704)

Curvature.

None

Question (1918 STEP I Q705)

Green's Theorem and its applications to Electrostatics.

Question (1918 STEP I Q706)

The potentials, charges, and energy of a system of conductors.

Question (1918 STEP I Q707)

Lines and tubes of electrostatic force, and equipotential surfaces.

Question (1918 STEP I Q708)

The parabolic motion of a particle under gravity.

Question (1918 STEP I Q709)

The conservation of momentum and energy; illustrate your account by considering the direct impact of spheres.

Question (1918 STEP I Q710)

The refraction of light, with applications to prisms and simple lenses.

Question (1920 STEP II Q701)

Homographic correspondence in Plane Geometry, with applications.

Question (1920 STEP II Q702)

Ruled surfaces, both developable and otherwise.

Question (1920 STEP II Q703)

Determinants.

Question (1920 STEP II Q704)

The employment of the Calculus of Residues

(a) in the expansion of functions,

(b) in the theory of equations,

(c) in the general theory of elliptic functions.

Question (1920 STEP II Q705)

Infinite integrals.

Question (1921 STEP III Q701)

The separation and approximate calculation of the real roots of algebraic equations.

Question (1921 STEP III Q702)

Discuss the general equation of the second degree in three dimensions, obtaining the necessary conditions for the various types of quadrics and degenerate quadrics.

Question (1921 STEP III Q703)

Moving axes as applied to the geometry of curves and surfaces.

Question (1921 STEP III Q704)

The uniform convergence of series.

Question (1921 STEP III Q705)

The theory of Riemann integration.

Question (1921 STEP III Q706)

Doubly periodic functions.

Question (1921 STEP III Q707)

Frobenius' method for the solution of differential equations. Illustrate your account by discussing fully Bessel's equation

$$x^2y'' + xy' + (x^2 - n^2)y = 0.$$

Question (1921 STEP III Q708)

Give a general account of the theorems connecting the Volume, Surface and Line integrals of mathematical physics, showing for example how they are applied in Electromagnetic and Hydrodynamical theory.

Question (1921 STEP III Q709)

Write a short account of the principal energy exchanges which occur during the production of a steady current by a voltaic cell or accumulator, during the charging of the latter, and during electrolytic decomposition of (say) water. Prove the following general theorems on steady currents in linear conductors.

(i) If $F = \Sigma C^2 R$, the summation being taken over all conductors in a network, and if no accumulation of charge is going on at any point, then the currents must actually be distributed so that $F - \Sigma EC_s$ is a minimum, E_s, C_s, R_s being the E.M.F., current and resistance of the sth conductor.

(ii) If a unit current is led into the network at A and out at B, the potential difference between C and D is the same as the potential difference between A and B when the unit current is led in at C and out at D. There is no cell producing an E.M.F. in any conductor.

(iii) If a unit E.M.F. exists in the conductor AB, the current produced in the conductor CD is equal to the current produced in the conductor AB by a unit E.M.F. in CD.

Question (1921 STEP III Q710)

The stability of floating bodies.

Question (1921 STEP III Q711)

The general theory of forces, couples and wrenches in three dimensions.

Question (1921 STEP III Q712)

The vibrations of uniform strings, or plane waves of sound.

Question (1921 STEP III Q713)

Establish Lagrange's equations of motion for a general dynamical system, including the form they reduce to for impulsive actions. Give proofs of the following general theorems:

(i) The reciprocal relation $\dot{q}_s/P_r = \dot{q}'_r/P'_s$.

(ii) Bertrand's Theorem that the energy generated by given impulses is diminished by the introduction of any constraint.

(iii) Kelvin's Theorem that the energy acquired when the system is started with prescribed velocities is increased by the introduction of any constraint.

Illustrate these theorems by discussing the motion of a waggon of mass M carrying a pendulum of mass m and length l which can swing in the direction of motion of the waggon.

Question (1913 STEP I Q801)

The theory of poles and polars, developed by projective methods.

Question (1913 STEP I Q802)

Continued fractions and their use for approximation to irrational numbers.

Question (1913 STEP I Q803)

The determination of the nature and position of the quadric represented by the general equation of the second degree

$$(a, b, c, d, f, g, h, u, v, w \gamma x, y, z, 1)^2 = 0.$$

Question (1913 STEP I Q804)

Give a proof of Cauchy's Theorem. Indicate some of its simplest applications (as for example in the theory of doubly periodic functions).

Question (1913 STEP I Q805)

Fourier's Series.

Question (1914 STEP I Q801)

Sketch a few typical applications of the concepts of (i) the 'line at infinity,' (ii) the 'circular points at infinity,' in pure and analytical geometry. Justify the use of these concepts against any objections you can imagine made to it.

Question (1914 STEP I Q802)

Prove (i) that any rational function can be expressed in the form

$$\Pi(x) + \sum_{\mu} \left\{ \frac{A_{\mu,1}}{x-\alpha_\mu} + \frac{A_{\mu,2}}{(x-\alpha_\mu)^2} + \cdots + \frac{A_{\mu,r_\mu}}{(x-\alpha_\mu)^{r_\mu}} \right\},$$

where $\Pi(x)$ is a polynomial and the α's and A's constants, (ii) that any real rational function can be expressed in the form

$$\Pi(x)+\sum_{\mu} \left\{ \frac{A_{\mu,1}}{x-\alpha_\mu} + \cdots + \frac{A_{\mu,r_\mu}}{(x-\alpha_\mu)^{r_\mu}} \right\} + \sum_{\nu} \left\{ \frac{B_{\nu,1}x + C_{\nu,1}}{x^2+2p_\nu x+q_\nu} + \cdots + \frac{B_{\nu,s_\nu}x + C_{\nu,s_\nu}}{(x^2+2p_\nu x+q_\nu)^{s_\nu}} \right\},$$

where $\Pi(x)$ is a real polynomial and the α's, A's, etc. real constants, and (iii) that each of these expressions is unique of its kind.

Question (1914 STEP I Q803)

The convergence of series of positive terms.

Question (1914 STEP I Q804)

Maxima and minima of functions of several variables.

Question (1914 STEP I Q805)

Ruled surfaces.

Question (1914 STEP I Q806)

Discuss the two methods (of harmonic analysis and of travelling waves) of representing at any time the transverse vibrations of tense strings, or the displacements in plane sound waves, after an arbitrary initial disturbance. In the case of the second method, consider reflection at a boundary.

Question (1914 STEP I Q807)

Obtain Lagrange's equations of motion, considering, in addition to the usual case, the forms appropriate to impulsive forces and to small vibrations of an elastic system about a position of stable equilibrium.

Question (1914 STEP I Q808)
Explain, with examples, the applications of 'conformal representation' to hydrodynamics and electricity.

Question (1914 STEP I Q809)
Describe the method of time determination by meridian observations of star transits, showing how our time system is based finally on the apparent motion of the sun. State the instrumental errors which have to be allowed for, and explain how this is done.

Question (1914 STEP I Q810)
Discuss, with the aid of Cotes's theorem and Helmholtz's formula, the properties of a system of thin coaxial lenses, and indicate the use of the cardinal points in geometrical constructions.

Question (1922 STEP II Q801)
Conics through four fixed points or touching four fixed lines.

Question (1922 STEP II Q802)
The analysis of vector fields.

Question (1922 STEP II Q803)
Limits and bounds of functions of a real variable.

Question (1922 STEP II Q804)
Mean-value theorems and Taylor's theorem.

Question (1922 STEP II Q805)
The Jacobian of n functions of n independent variables.

Question (1922 STEP II Q806)
Partial differential equations.

Question (1922 STEP II Q807)
Spherical harmonics or Fourier series.

Question (1922 STEP II Q808)

Develop the formulae expressing the acceleration of a point in terms of its coordinates referred to moving rectangular axes. Use these formulae to find the equations of motion of a non-viscous fluid referred to cylindrical coordinates.

Question (1922 STEP II Q809)

The application of conjugate functions to the solution of problems in electrostatics or current electricity.

Question (1922 STEP II Q810)

State Kepler's three laws concerning the orbits of planets and shew how they are related to the theory of gravitation.

Question (1922 STEP II Q811)

Trace the steps by which the equations of motion of a system of particles are derived from the Newtonian equations applicable to the individual particles of which the system is composed. It may be assumed that the reaction between two particles is in the line joining them. Hence develop equations connecting the external forces with the kinetic energy of rigid and non-rigid bodies.

Question (1922 STEP II Q812)

The first and second laws of thermodynamics.

Question (1922 STEP II Q813)

Prove the theorem that the circulation round a given circuit of particles in a non-viscous fluid is constant during the motion of the fluid. Hence develop the theory of irrotational motion.

Question (1923 STEP III Q801)

The invariants of a system of two conics.

Question (1923 STEP III Q802)

Envelopes of plane curves.

Question (1923 STEP III Q803)

Curvilinear coordinates.

Question (1923 STEP III Q804)

Differentials.

Question (1923 STEP III Q805)

Series of complex constants.

Question (1923 STEP III Q806)

Give the theory of the reduction of a three dimensional system of forces, and the various conditions for the equilibrium of such a system. Prove that a line distribution of couple of amount H per unit length of a plane closed curve s, the axis of the couple at any point being normal to, and in the plane of the curve, is statically equivalent to a line distribution of force of amount $-\dfrac{\partial H}{\partial s}$, the direction of the force at any point being at right angles to the plane of the curve.

Question (1923 STEP III Q807)

Discuss the theory of the small oscillations of a dynamical system which is slightly disturbed from a position of stable equilibrium.

Question (1923 STEP III Q808)

The stability of floating bodies.

Question (1923 STEP III Q809)

Define the coefficients of potential, capacity and induction of a system of conductors, and give an account of their properties. Find the electrical energy of such a system, and prove that it is diminished by the introduction of a new conductor.

Question (1923 STEP III Q810)

Prove that

$$\iint_S (lu + mv + nw)d\sigma = \iiint_T \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) dxdydz,$$

where l, m, n are the direction cosines of the outward drawn normal to the boundary S of T, and give some of the applications of this result either in electrostatics or in the theory of the irrotational motion of a liquid.

Question (1923 STEP III Q811)

Give the theory of two dimensional surface waves on a liquid under no force but gravity, considering in particular waves at the common surface of two liquids.

Question (1924 STEP III Q801)

Pencils and ranges of conics, and their relation to the theory of confocal conics.

Question (1924 STEP III Q802)
Curvature of surfaces.

Question (1924 STEP III Q803)
Continued fractions.

Question (1924 STEP III Q804)
The Riemann integral.

Question (1924 STEP III Q805)
Newtonian potentials.

Question (1924 STEP III Q806)
Series of variable terms, in particular power series.

Question (1924 STEP III Q807)
The proof of Cauchy's theorem.

Question (1924 STEP III Q808)
Graphical methods in statics and their geometrical applications.

Question (1924 STEP III Q809)
Theorems on the changes in motion of a system of bodies produced by impulses.

Question (1924 STEP III Q810)
The "instantaneous ellipse" for a particle moving under a gravitational force to a fixed centre and a perturbing force.

Question (1924 STEP III Q811)
The motion of a top.

Question (1924 STEP III Q812)
The "uniqueness theorems" of electrostatics and their applications in the methods of images and of inversion.

Question (1924 STEP III Q813)

The properties of the velocity-potential ϕ and the stream-function ψ in the hydrodynamics of an incompressible fluid in two and three dimensions.

Question (1924 STEP III Q814)

The porous-plug experiment and the determination of absolute temperature.