Question (1976 STEP II Q9)

The following is from an advertisement for 'X' beer. We've tried our famous 'X' Taste Test on twenty beer experts, pouring three glasses, one from the tap, one from the can, and one from the bottle. And then we've asked which is which. Result? No one identified the three correctly. Why? Because all three glasses have the same famous 'X' Taste. What confidence can you have in the reasoning in this advertisement?

Question (1977 STEP II Q9)

Do you think that the following deductions are correct? Explain your reasons simply but clearly. (i) The average age at death of generals is considerably higher than that for the whole population. This shows that generals take care not to expose themselves to danger. (ii) I have tossed this coin twice and it came down heads each time. Therefore it is probably an unfair coin. (iii) I have tossed this coin 1000 times and it came down heads 276 times. Therefore it is probably an unfair coin.

Question (1979 STEP II Q7)

The random variable C takes integral values in the range -5 to 5, with probabilities

$$\Pr[C = -5] = \Pr[C = 5] = \frac{1}{20}; \quad \Pr[C = i] = \frac{1}{10} \quad (-4 \leq i \leq 4).$$

Calculate the mean and variance of C. A shopper buys 36 items at random in a supermarket, and, instead of adding up his bill exactly, he rounds the cost of each item to the nearest 10p, rounding an odd 5p up or down with equal probability. Should he suspect a mistake if the cashier asks him for 20p less than he had estimated?

Question (1969 STEP III Q2)

A Bernoulli trial results in success with probability p or failure with probability $1 - p$. If X is the number of successes in n independent Bernoulli trials show that $E(X) = np$ and $\text{var}(X) = npq$. We wish to estimate by means of a random sample the proportion p of the population of a certain large city who are cat lovers. How large should the sample be if we wish to be 95% certain that the error in estimating p will be less than 0.01?

Question (1975 STEP III Q10)

At a certain university, two lecturers (A and B) each gave parallel courses in first-year analysis and in second-year algebra, on the same syllabus in each case, and students were free to choose which lecturer they followed. One day a count was made, and attendances were found to be as in the following table:

	A	B
Analysis	95	25
Algebra	55	25

Test independently of each other the following propositions:

(i) that A and B are equally popular for analysis,

(iii) that A and B are equally popular for algebra,

(iii) that their popularities are independent of the course they are giving.

Compare your answers to (i) and (ii) with that of (iii), and comment.

Question (1981 STEP III Q10)

The Royal Mint wishes to determine whether a given coin is fair or not, and has decided to conduct the following test. The coin will be tossed N times, and will be deemed fair if X, the proportion of heads, lies in the interval (a, b). Let p be the (unknown) probability of obtaining heads on a single toss of the coin. N, a, b must satisfy the conditions: (i) If $p = 0.5$, $P(a \leq X \leq b) \geq 0.95$. (ii) If $p \geq 0.51$, $P(X > b) \geq 0.95$. (iii) If $p \leq 0.49$, $P(X < a) \geq 0.95$. Find the approximate value of the smallest possible value of N, and appropriate values of a and b for this N. The experiment was conducted with $N = 40,000$. State what conclusions may be drawn from (a) 20,400 heads, (b) 19,900 heads.

Question (1977 STEP IIII Q11)

The author of a scientific paper claims to have done the following experiment 3600 times. The subject wrote down a number, then a die was thrown and the number shown on the die compared with the prediction. He claims that the results were as shown.

Number on die	1	2	3	4	5	6	Total
Number of times thrown	604	425	694	664	495	718	3600
Number of correct predictions	95	84	181	171	114	203	848

(a) If the experimental data is not fraudulent does it provide good evidence that the future may be predicted? (b) Looking at the table, do you find evidence that the data is fraudulent? If so, explain why.