Question (1971 STEP II Q6)

A hospital buys batches of a certain tablet from a pharmaceutical company. A tablet is considered unsatisfactory if it contains more than 1 microgram of arsenic. It is known that within any batch of tablets the arsenic content is normally distributed with standard deviation 0.05 micrograms about a mean which depends on the batch. From every batch the hospital randomly selects n tablets for analysis, and rejects the batch if the mean arsenic content of the n tablets is greater than C. What values should be chosen for n and C if the desired chances of rejecting batches with 0.1% and 1% of defective tablets, respectively, are 20% and 90%?

Question (1972 STEP II Q15)

A firm needs to buy a large number of metal links which must stand a load of 1.20 tons weight. There are two grades on the market, the load under which a link will break being in each case normally distributed with parameters as follows:

	Mean	Standard deviation
Grade 1	1.20	0.10
Grade 2	1.60	0.20

Grade 1 cost 80p each, grade 2 cost £1.00 each, and it costs the firm £10.00 in damaged equipment each time a link breaks. Which grade should the firm buy? Grade 3 now comes on the market at 95p. The critical load is again normally distributed with standard deviation 0.10, but the mean is unknown. Testing to destruction ten of these, chosen randomly, gives figures for the critical load:

1.17 1.29 1.26 1.31 1.55 1.36 1.42 1.13 1.32 1.29

Making use of a suitable 95% confidence limit, what advice would you give the firm as to whether or not they should buy the new grade?

Question (1973 STEP II Q9)

An anthropologist encounters a large group of savages in the jungle. He knows that either they all come from tribe A or they all come from tribe B. In both cases their heights are independently distributed; if they are from A then the heights are normal with mean $\mu_A = 60$ inches and standard deviation $\sigma = 5$ inches; if they are from B the heights are normal with mean $\mu_B = 66$ inches, and standard deviation $\sigma = 5$ inches. In order to decide to which tribe they belong, the anthropologist uses a rule of the following form. He assigns them to A if $\overline{x}_n < \xi$, and otherwise to B, where $\overline{x}_n$ is the mean of the heights of n savages. Show how he should choose ξ in order that α, the probability of wrongly assigning them to B, is 0.05. Find the corresponding value of β, the probability of wrongly assigning them to A, and find how large n should be in order that β is 0.01 or less. [You may assume that $\overline{x}_n$ has a normal distribution, whose mean depends on whether the savages are from A or B.]

Question (1974 STEP II Q9)

In a sample of 50 male undergraduates at Cambridge in 1900 the mean height was found to be 68.93 in. In a sample of 25 male undergraduates at Cambridge in 1974 the mean height was 70.66 in. It may be assumed that the heights of male undergraduates are always normally distributed with a standard deviation of 2.5 in. Is it reasonable to suppose that there has been an increase in the average height of male undergraduates at Cambridge over the past 74 years? Explain carefully the reasoning you use.

Question (1975 STEP II Q9)

An entomologist measures the lengths of 8 specimens of each of two closely related species of bees. His measurements of species A and of species B have mean values 15 mm and 17 mm respectively. If he believes that in each species length is normally distributed with standard deviation 2 mm, should he conclude that the mean lengths of the two species differ? What procedure should he use if he does not know the standard deviation (though still believing it to be the same for both species)?

Question (1978 STEP II Q8)

A tug-of-war contest is to be held between two colleges. The weights of students in College A follow a normal distribution with mean 140 lb and standard deviation 8 lb. Thanks to the superiority of its kitchens, the weights of students in College B follow a normal distribution with mean 150 lb and standard deviation 6 lb. Teams are chosen by selecting n students at random from each college. How large must n be in order to ensure that with probability at least 0.9 the combined weight of the College B team exceeds that of the College A team by at least 50 lb?

Question (1979 STEP II Q8)

An experiment was conducted to investigate the effect of a new fertilizer on the yield of tomato plants. Ten plants were grown using the new fertilizer, and ten using the one previously recommended, giving yields (in kg): New 1.5 1.9 1.7 1.8 1.5 2.0 2.0 1.8 1.9 1.8 Old 1.4 1.3 1.3 1.5 1.8 1.3 1.1 1.3 1.4 1.6 Assuming that the yields are normally and independently distributed, with means μ_N for plants having the new fertilizer and μ_0 for those having the old one, and with standard deviation 0.3 kg whichever fertilizer was used, test whether or not there is evidence that the new fertilizer is an improvement on the old one. How would you estimate the standard deviation of the yield of a tomato plant if it was not known to be 0.3 kg?

Question (1980 STEP II Q7)

The average weight in grams of the contents of a sachet of instant mashed potato varies between batches, because of the variable quality of the synthetic feedstock. Within a given batch, the weights of the sachets are independently and normally distributed, with common unknown mean m and standard deviation $0 \cdot 1$. In order to check the weight of a given batch, the manufacturer weighs the contents of 25 sachets, obtaining an average weight of $4 \cdot 92$. Does this give him good grounds for rejecting the hypothesis that m is really 5? He now decides upon the policy of rejecting a batch whenever the average weight of a sample of N sachets falls below T. If N and T are to be chosen so that the probabilities of wrongly rejecting a batch with $m = 5$ and of wrongly accepting a batch with $m = 4 \cdot 95$ are both less than $0 \cdot 05$, what values would you choose to make N as small as possible?

Question (1967 STEP III Q12)

A manufacturer is asked to supply steel tubing in lengths of 10 feet. Several samples are obtained from him and the mean lengths in feet of four samples each of 16 tubes found to be as follows:

$$10 \cdot 16; \quad 10 \cdot 38; \quad 10 \cdot 31; \quad 10 \cdot 07.$$

What type of distribution would you expect mean lengths such as these to have and why? Samples are also obtained from another source, and in this case the mean lengths in feet of five samples of 16 tubes are found to be as follows:

$$10 \cdot 15; \quad 10 \cdot 36; \quad 10 \cdot 11; \quad 10 \cdot 11; \quad 10 \cdot 07.$$

Assuming that both manufacturers produce tubing whose length has a standard deviation of $0 \cdot 48$ feet, is there any evidence that either manufacturer's tubing has a mean length greater than $10 \cdot 1$ feet? Is there any evidence that tubes supplied by the two manufacturers differ in mean length? [Let

$$\Phi(X) = \int_{-\infty}^{x} \phi(x) dx,$$

where $\phi(x) = (2\pi)^{-1} \exp(-\frac{1}{2}x^2)$. Then $\Phi(-2 \cdot 58) = 0 \cdot 005$, $\Phi(-2 \cdot 33) = 0 \cdot 01$, $\Phi(-1 \cdot 96) = 0 \cdot 025$, $\Phi(-1 \cdot 64) = 0 \cdot 05$.]

Question (1968 STEP III Q4)

For a certain mass-produced item the time that a randomly chosen individual lasts before failure may be supposed for practical purposes to be Normal with mean 100 and variance 1. A slight change is made in the conditions of manufacture, and the times until failure of n independently chosen items fail are determined, these being $x_1, x_2, \ldots, x_n$. Construct a significance test at the 5% level which would be appropriate in order to discover whether the mean length of life has increased, and explain carefully the meaning of such a procedure. (The variance may be supposed unchanged.) Determine how large n must be in order that the probability of not rejecting the null hypothesis is 0.05 if in fact the new mean is 101.

Question (1970 STEP III Q3)

Explain what is meant by the term 'standard error of the mean'. Matches are put into a box five at a time until the weight of the box and matches combined reaches M grams, when the box is said to be full. The weight of an individual match is normally distributed with mean m grams and standard deviation σ grams. The weight of an empty match-box is normally distributed with mean $5m$ grams and standard deviation 2σ grams. Find the value of M such that there is only one chance in a hundred that a full match-box contains fewer than 50 matches.

Question (1970 STEP III Q4)

Two normal distributions have different means of 100 and 110 cm and the same standard deviation of 10 cm. A random sample is to be drawn from one of these distributions on the basis of which we have to decide which distribution is being sampled. We wish to have less than 1% probability of making an error if the distribution is really the one with mean 100 and less than 5% probability of error in the other case. What is the smallest possible size of sample?

Question (1971 STEP III Q11)

The following figures are the additional hours of sleep gained by the use of a certain drug on ten patients:

$$+1.9,\ +0.8,\ +1.1,\ +0.1,\ -0.1,\ +4.4,\ +5.5,\ +1.6,\ +4.6,\ +3.4.$$

(i) Using a significance test discuss whether these results show convincingly that the drug is an effective sleeping pill.

(ii) In what circumstances is the test you have used valid?

Question (1980 STEP III Q10)

Let $X_1, X_2, ..., X_n$ be a random sample of size n drawn from a normal distribution with variance 1 and with unknown mean β. Show how to use the sample mean to construct an interval which contains β with probability approximately 0.95. Now suppose that $X_1, X_2, ..., X_n$ are not necessarily normally distributed, but merely that their common unknown distribution is continuous (so that $P[X_i = x] = 0$ for any real x). Show that, if q_α is the α-quantile of the unknown distribution (i.e. if q_α is such that $P[X_i \le q_\alpha] = \alpha$), and if $X_{(1)}, X_{(2)}, ..., X_{(n)}$ denotes the sample $X_1, X_2, ..., X_n$ arranged in ascending order, then $P[X_{(r)} < q_\alpha < X_{(r+1)}] = \binom{n}{r}\alpha^r(1-\alpha)^{n-r}$. Use this fact to construct, in the case when $n = 6$, an interval within which the median $q_{1/2}$ of the distribution will lie with probability at least 0.95. Evaluate both intervals when $(X_{(1)}, X_{(2)}, ..., X_{(6)}) = (-0.92, -0.77, 0.41, 0.47, 0.48, 0.99)$.

Question (1982 STEP III Q10)

The King of Smorgasbrod proposes to raise lots of money by fining those who sell underweight kippers. The weight of a kipper is normally distributed with mean 200 grams and standard deviation 10 grams. Kippers are packed in cartons of 625 and vast quantities of them are consumed. The Efficient Extortion Committee has produced three possible schemes for determining the fines.

1. Weigh the entire carton, and fine the vendor 1500 crowns if the average weight of a kipper is less than 199 grams.

2. Weigh 25 randomly selected kippers and fine the vendor 100 crowns if the average weight of a kipper is less than 198 grams.

3. Remove kippers one at a time and at random from the carton until an overweight kipper has been found, and fine the vendor $3n(n-1)$ crowns, where n is the number of kippers removed.

Which of the EEC's schemes should the avaricious king select?

Question (1970 STEP IIII Q8)

The number of hours of sleep of a group of patients was recorded. On a subsequent night the patients were each given a sleeping pill and the number of hours of sleep was again recorded. The results were as follows:

Patient number	1	2	3	4	5	6	7	8	9	10
Hours, before treatment	7.0	6.1	6.0	3.0	2.7	3.2	4.1	7.1	0.1	2.6
Hours, after treatment	7.2	5.9	6.0	6.2	4.1	3.5	4.7	7.0	0.5	3.5

The results show that most patients slept better after taking the sleeping pill. Are the figures sufficient to demonstrate beyond reasonable doubt that the pill is effective? Justify the use of any statistical technique you have employed.

Question (1981 STEP IIII Q11)

In the run up to the general election in Ruritania, two polling organisations, A and B, attempted to measure the support of the two political parties, the Reds and the Blues, by each questioning a random sample of 1000 voters (out of a population of several million). The combined results were

Polled 2000 Red supporters 1056 Blue supporters 944

What conclusions may be drawn from these figures? A few days after these figures were published, it was discovered that A and B had each subcontracted to polling organization C the task of sampling 500 voters in the A–L section of the alphabet, and had sampled the remaining 500 from the M–Z section itself. To cut costs, C had given the results of the same sample of 500 to A and B. Is it possible to deduce anything from the figures now?