Question (1978 STEP II Q12)

Two particles of masses m and $2m$ are suspended over a movable pulley of mass m by a light string of length l. The movable pulley is itself connected to a particle of mass $4m$ by a light string of length L which passes over a fixed pulley. Find the acceleration of the particle of mass $4m$. [You may neglect the moments of inertia of the pulleys.]

Question (1966 STEP III Q7)

A smooth pulley is fixed to the edge of the roof of a building at a height h from the ground. A light cord of length l is passed over the pulley and has two buckets attached to its ends, one of which rests on the ground and one of mass $3m$ hanging. A man on the roof drops a brick of mass m into the second bucket and it remains in the sand without bouncing. Find the impulsive tension in the cord. Which of the buckets will hit the ground next, and after how long?

Question (1952 STEP III Q406)

A light string passes over a small smooth fixed pulley and to one end is attached a mass M and to the other a second small light pulley over which passes a second string carrying masses m_1 and m_2 at its ends. Find the condition that if the system is released from rest the mass M will not move, and determine the total downward force on the fixed pulley.

Question (1915 STEP I Q101)

A smooth sphere is suspended from a fixed point by a string of length equal to its radius. To the same point a second string is attached which after passing over the sphere supports a weight equal to that of the sphere. Shew that the string which supports the sphere makes an angle $\sin^{-1}(\frac{1}{4})$ with the vertical.

Question (1917 STEP I Q109)

Two particles of masses m and $3m$ are connected by a fine string passing over a fixed smooth pulley. The system starts from rest and the heavier particle, after falling 8 feet, impinges on a fixed inelastic support. Find the velocity with which it is next jerked off the support; and shew that the system finally comes to rest 3 seconds from the beginning of the motion.

None

Question (1924 STEP I Q109)

Two equal flat scale pans are suspended by an inextensible string passing over a smooth pulley so that each remains horizontal. An elastic sphere falls vertically and when its velocity is u it strikes one of the scale pans and rebounds vertically. Show that the sphere takes the same time to come to rest on the scale pan as it would if the scale pan were fixed.

Question (1935 STEP I Q109)

An Atwood's machine consists of a light frictionless pulley carrying a light string at one end of which is carried a mass A of 19 ounces, and at the other end of which is carried a mass B of 17 ounces and a rider of 4 ounces. The system is released from rest with the rider at a height of 100 inches above a fixed ring through which B ultimately passes and on which the rider is removed. Shew that the mass B comes to rest at a depth of 90 inches below the ring, and that it then ascends, picks up the rider and comes to rest once again at a height 81 inches above the ring.

Question (1941 STEP I Q110)

A uniform circular disc of radius a and mass M can turn in its own plane about a fixed horizontal axis through the centre. A light inextensible string lies in a rough groove in the edge of the disc; the end portions of the string are vertical and masses $M, 2M$ are carried at the ends. If these masses move vertically, show that the angular acceleration of the disc is $2g/7a$. Find also the tensions in the vertical portions of the string. Find the magnitude of the frictional couple that must be applied to the disc in order that its angular acceleration may be $g/7a$.

Question (1937 STEP I Q108)

Masses $m_1, m_2, \dots m_n$ are attached to points of a light inextensible string which hangs in equilibrium, suspended by its two ends. If the lengths of the segments of the string are given, together with the relative positions of the two ends of the string, show how to obtain sufficient equations to determine the inclinations of the segments and the tensions in them. If the masses are each equal to m and are attached at equal horizontal intervals h, show that the points of attachment lie on a parabola of latus rectum $2hT_0/mg$, where T_0 is the horizontal component of the tension. Show also that an equal parabola touches the segments of the strings at their middle points, and that the distance between the vertices of these two parabolas is $mgh/8T_0$.

Question (1921 STEP I Q206)

A string passes over a smooth fixed pulley and to one end there is attached a mass M_1, and to the other a smooth light pulley over which passes another string with masses M_2 and M_3 at the ends. If the system is released from rest show that M_1 will not move if

$$\frac{4}{M_1} = \frac{1}{M_2} + \frac{1}{M_3}.$$

What is the pressure on the fixed pulley?

Question (1922 STEP I Q206)

A mass M rests on a smooth table and is attached by two inelastic strings to masses m, m' $(m' > m)$, which hang over smooth pulleys at opposite edges of the table. The mass m' falls a distance x from rest, and then comes into contact with the floor (supposed inelastic). Shew that m will continue to ascend through a distance y given by

$$\frac{y}{x} = \frac{(m' - m)(M + m)}{m(M + m + m')}.$$

Shew further that when m' is jerked into motion again as m falls it will ascend a distance $x(M + m)^2/(M + m + m')^2$.

Question (1925 STEP I Q203)

Define mechanical advantage and efficiency. Shew that the mechanical advantage in the pulley system shown is twice the efficiency, the radii of the pulleys being a and $a/2$, and A being a fixed point directly below the axis of the upper pulley. The lower pulley runs smoothly on its bearings, whereas the rotation of the upper pulley is opposed by a frictional couple proportional to the pressure of the pulley on its bearings. If E is the efficiency, shew that the least force P which will just prevent the weight from slipping down is $\frac{1}{2}EW$. (Neglect the weight of the upper pulley and assume that the rope does not slip.)

Question (1942 STEP I Q206)

A light rope hangs over a light pulley. A mass M is attached to one end of the rope and a man of mass M clings to the other end of the rope. The man then climbs the rope with a uniform velocity v relative to the rope. Describe the ensuing motion. Explain how energy and linear momentum are conserved in this case.

Question (1925 STEP IIII Q207)

A uniform thin hollow right circular cylinder stands upright on a table, and three smooth equal spheres each of weight w are placed inside it. The ratio of the radius of a sphere to that of the cylinder is α. Prove that if $\frac{1}{2} > \alpha > 2\sqrt{3} - 3$, so that two of the spheres rest upon the ground, the cylinder will not overturn if its weight exceed $\frac{w}{2}(1 + \sqrt{1 - 2\alpha^2})$. Each sphere is to be taken in contact with the cylinder and with the other two spheres.

Question (1935 STEP I Q310)

Two unequal masses M_1 and M_2 are joined by a light inextensible string slung over a heavy, rough and freely pivoted pulley whose moment of inertia is Mk^2, and radius is a. $M_1 > M_2$. Shew that if μ is the coefficient of friction between the string and the pulley, slipping will occur when the system is released from rest unless

$$\mu > \frac{1}{\pi}\log_e \frac{M_1(2M_2 + M\frac{k^2}{a^2})}{M_2(2M_1 + M\frac{k^2}{a^2})}.$$

If slipping does occur, shew that the acceleration of the masses is

$$\frac{g(M_1 - M_2e^{\mu\pi})}{M_1 + M_2e^{\mu\pi}},$$

and that the angular acceleration of the pulley is

$$\frac{M_1M_2(e^{\mu\pi} - 1)}{2agMk^2(M_1 + M_2e^{\mu\pi})}.$$

Question (1914 STEP II Q307)

A mass M is fastened to one end of a fine string which passes over a smooth pulley, and to the other end of the string is attached a smooth pulley: over this second pulley a fine string passes, one end of which is fastened to the ground and the other end to a mass m. Determine the motion, and shew that if $M = 4m$ the tension of the string passing over the fixed pulley is $3mg$.

Question (1918 STEP III Q306)

State Newton's Second Law of Motion and shew how it leads to the equation $P = mf$. A pulley of mass m is connected with a mass $4m$ by a string which hangs over a fixed smooth pulley; a string with masses m and $2m$ at its extremities is hung over the pulley. If the system is free to move, find the acceleration of each of the masses.

Question (1926 STEP III Q307)

A smooth ring of mass M is threaded on a light flexible string which is then hung over two smooth fixed pulleys. Masses m and m' are tied to the ends of the string, the ring is on the portion of string between the pulleys, and the various portions of the string hang vertically. The system is now released. Find the acceleration of the ring, and prove that it is zero if

$$M = \frac{4mm'}{m + m'}.$$

Question (1938 STEP III Q310)

Two particles A and B each of mass m are connected by a light inextensible string of length l which passes through a small hole at a point O in a smooth horizontal table on which the particle A can move while B hangs vertically. The particle B is attached by a light elastic spring to a fixed point which is at a distance $3l/2$ vertically below O. The elastic spring has a natural length l and modulus of elasticity $2mg$. Initially the string AB is tight and B is released from rest while simultaneously A is projected horizontally with a velocity $\sqrt{(2gl)}$ at a distance $l/2$ from O and at right angles to OA. Show that the mass B is next instantaneously at rest when it has moved through a distance $l/4$.

Question (1919 STEP IIII Q306)

Two weights A and B are connected by a string passing over a smooth light pulley. To the weight B is attached another weight C by a string of length 2 ft. B and C are held initially in contact and resting on a platform vertically below the pulley. If the masses of A, B and C are 5, 3 and 4 lb. respectively, shew that when the system is free to move, the weight C will strike the platform again after $12/\sqrt{g}$ seconds and that the weight B will come momentarily to rest at a distance $1\frac{1}{2}$ ft from the platform.

Question (1938 STEP I Q406)

Discuss the absolute and gravitational units of force and the relations between them. Two pans each of mass m are connected by a light inextensible string passing over a smooth pulley and the pans hang freely in equilibrium. A uniform chain of length l and mass m is held over one pan with its lower end just touching it. If the chain is released from rest, find the time that elapses before it is all coiled up in the pan, neglecting the finite size of the coil produced.

Question (1923 STEP III Q407)

A string passing over a smooth pulley carries a mass $4m$ at one end and a pulley of mass m at the other. A string carrying masses m and $2m$ at its ends passes over the latter pulley. Find the acceleration of the mass $4m$ when the system is moving freely under gravity.

Question (1927 STEP III Q407)

Two masses, m_1 and m_2 lb., are connected by a light elastic string passing over a smooth pulley. The string stretches one foot under a tension of P poundals. The masses are supported so that the two sides of the string are vertical and just slack, and the mass m_1 is then released. Prove that the mass m_2 will begin to rise after a time

$$\sqrt{\frac{m_1}{P}}\cos^{-1}\left(1-\frac{m_2}{m_1}\right).$$

Question (1917 STEP IIII Q408)

Two weights W, W' balance on any system of pullies with vertical strings. If a weight w be attached to W, shew that it will descend with acceleration

$$g/\left[1+\frac{W(W+W')}{wW'}\right],$$

neglecting the inertia of the pullies.

Question (1926 STEP III Q510)

Two unequal masses are connected by a string of length l which passes through a fixed smooth ring. The smaller mass moves as a conical pendulum while the other mass hangs vertically. Find the semi-angle of the cone, and the number of revolutions per second when a length a of the string is hanging vertically.

Question (1915 STEP III Q605)

One end of a light string is fixed, and the string, hanging vertically in a loop in which a ring of mass m moves, passes over a smooth fixed pulley and has a mass M tied to the other end. Shew that in the motion the tension of the string is $\frac{3Mmg}{4M+m}$.

Question (1926 STEP III Q613)

Two particles m and m' are connected by a string of length l and rest on a smooth horizontal table with the string taut. If m is just at the edge of the table and is allowed to fall, determine the motion of m and m' after m' is drawn over the edge of the table, and shew that the system will rotate with uniform angular velocity $\sqrt{\{2gm/l(m+m')\}}$.

Question (1930 STEP III Q610)

Hanging over a smooth pulley are two scale pans A and B. A is of mass m, and in it is an insect of mass $(n-1)m$. B is of mass $2m$, and in it is an insect of mass $(n-2)m$. When the system is in equilibrium the insect in B jumps vertically with velocity c relative to the pan. Find the initial velocity of the insect and also the time that elapses before the insect returns to the pan, and prove that the system is then in its initial position.

Question (1919 STEP III Q709)

A string passing over a smooth fixed pulley carries a mass $2m$ at one end and another smooth pulley of mass m at the other end. A string having masses m and $2m$ at its ends passes over the second pulley. If motion of the system takes place under gravity, find the accelerations of the various parts of the system and the acceleration with which the centre of gravity of the whole system descends.

Question (1923 STEP III Q713)

A string, of which one end is attached to a mass m lying on a smooth table, passes over the edge of the table, and after passing over a smooth fixed pulley close to the table and on a level with it has its other end attached to a mass m'; between the table and the pulley the string hangs in a loop and supports a smooth ring of mass M. The string lies in a vertical plane perpendicular to the edge of the table. Find the motion and the tension of the string, and shew that the mass m' will remain at rest if $M = 4mm'/(2m - m')$.