Question (1981 STEP II Q8)

Let $X_1, ..., X_m$ be independent normally distributed random variables, with mean μ and variance σ^2. Let $X > 0$, and let Y be the number of observations falling in the range $(a - X, \mu + X)$. Give an expression for $P(Y = r)$ for $r = 0, 1, ..., m$. If $\alpha = \frac{1}{2}$ and $m = 10$, what is $P(Y \leq 2)$? (You may leave your answer in a form suitable for calculation.)

Question (1984 STEP II Q9)

Let $X_1, X_2, ...$ be independent random variables uniformly distributed on $[1, 2]$. Show that
$$\Pr(a < (X_1 X_2 ... X_n)^{1/n} < b) \to 1 \text{ as } n \to \infty$$
if and only if $a < 4/e < b$. You may use any results from probability theory that you know.

Question (1980 STEP IIII Q10)

A businessman puts money into one deal each year. At the end of each year, the deal has either fallen through, in which case he loses his entire outlay, or has been successful, in which case he recovers twice his outlay. The outcomes of different deals are independent, each one having a chance $p > \frac{1}{2}$ of success, and the businessman can choose how much money he wishes to risk each year, within the limits of his current fortune. A possible strategy for the businessman is to put his entire current fortune into each deal. If his initial fortune is X_0, show that his fortune X_n after n years' dealing has expectation $(2p)^n X_0$, but that he is certain to lose all his money eventually. A more conservative strategy is for him to invest a fixed proportion s of his current fortune in each deal, where $0 < s < 1$. Show that the expected value of X_n is now only $[1 + s(2p - 1)]^n X_0$, but that, by choosing s suitably, he can guarantee an eventual compound growth rate of $\alpha = 2p^p(1 - p)^{1-p}$, in the sense that $\frac{1}{n} \log_e(X_n/X_0)$ tends to $\log_e \alpha$ as $n \to \infty$. [Hint. Show that, with an appropriate definition of Y_n, X_{n+1} can be expressed in the form $X_{n+1} = X_n(1 + s)^{Y_n}(1 - s)^{1-Y_n}$. You may assume the 'law of large numbers' in the form: if $Y_1, Y_2, \ldots$ are independent and identically distributed random variables with mean β, then $\frac{1}{n} \sum_{j=1}^{n} Y_j \to \beta$ as $n \to \infty$.]