Question (1982 STEP I Q12)

A jar contains r red, b blue and w white sweets. A greedy child picks out sweets one by one at random and eats them, until only sweets of a single colour remain. Show, by induction or otherwise, that the probability that only red sweets remain is $\frac{r}{r+b+w}$.

Imagine we continue pulling out sweets. Then the last sweet to be removed will be the same colour as all the other sweets which are the same colour.

Therefore the question is equivalent to "what is the probability the last sweet is red", but since all possible orders of sweets are equally likely, this is just $\frac{r}{r+b+w}$

Question (1983 STEP I Q12)

A player deals cards from a pack of 52 in sets of four. The first set of four consists of cards of different suits. What is the probability that the last set of four consists of cards of different suits? Had the first set of four consisted of cards of the same suit, what would the probability have been that the last set of four were also of one suit?

Question (1984 STEP I Q7)

Two Oxford undergraduates, Algy and Berty, resolve to duel with champagne corks at twenty paces. Each shot that Algy fires has a probability 1/4 of hitting Berty and each shot that Berty fires has a probability 1/5 of hitting Algy. The supply of corks is unlimited. What are Algy's chances of escaping without being hit if (i) they fire alternately starting with Algy until one of them is hit? (ii) they fire alternately starting with Berty until one of them is hit? (iii) they fire simultaneously and repeatedly until one or both is hit?

Question (1974 STEP II Q7)

A box contains b black and r red balls. Balls are drawn from it at random one at a time. After each draw the drawn ball is replaced and c balls of its colour are added to the box. Prove by induction or otherwise that the probability $p(n)$ that a black ball is drawn on the nth occasion is $b/(r+b)$. What is the expected number of black balls in the box immediately before the $(n+1)$th draw?

Question (1978 STEP II Q5)

Each week, a boy receives pocket money only on condition that he wins two games in a row when playing three successive chess games with his father and mother alternating as opponents. The boy knows that his mother's probability of winning is $\frac{3}{4}$, but his father's probability of winning is only $\frac{1}{2}$. To maximise his chance of winning two games in succession, should he play the sequence father-mother-father, or mother-father-mother? Assuming that each week the boy plays the sequence more favourable to him, what is the expected number of weeks between two successive occasions on which he receives pocket money?

Question (1979 STEP II Q6)

Under Atypical Tennis Players rules, a game is won when either player has scored two more points than his opponent. If the chance of the first player winning any given point is p, independently of the outcomes of all other points, evaluate the probability $f(p)$ that he wins the game. Show that, for $\frac{1}{2} < p < 1$, $f(p) > p$, and that, for $p > \frac{1}{2}$, $f(p) - \frac{1}{2} \geq 2(p - \frac{1}{2})$.

Question (1972 STEP III Q17)

A and B play a series of games the results of which are independent. In each game, A has probability p of winning, B probability q, where $q = 1 - p$, and each pays the winner one unit. Supposing that A starts with n units and B with N, let c_n denote the probability that A loses all his money. By writing down a relation between c_{n+1}, c_n and c_{n-1}, show that

$$c_n = \frac{(q/p)^N - (q/p)^n}{(q/p)^N - 1} \quad (p \neq q)$$

and

$$c_n = 1 - (n/N) \quad (p = q).$$

A casino can be considered as an opponent with infinite reserves of capital. Find the probability that a compulsive gambler A, with only a finite amount of capital, will eventually lose all his money to the casino, and say whether British establishments $(p = q)$ differ in this respect from those on the Continent $(p < q)$.

Question (1977 STEP III Q9)

A table tennis championship is arranged for 2^n players. It is organised as a 'knockout' tournament with a draw for opponents before each round except the final. (Only the winners of a round proceed to the next.) Two players are chosen at random before the draw for the first round. What are the probabilities that they meet (i) in the first round? (ii) in the final? (iii) in any round of the tournament?

Question (1974 STEP IIII Q10)

In one game of a tennis match the probability that a player serving wins any particular point is $\frac{3}{4}$. What is the probability that the player serving wins the game? [The game finishes as soon as one player has won at least four points and is at least two points ahead of his opponent.]