Question (1974 STEP II Q11)

Two identical small smooth spheres S_1 and S_2 of radius b are free to slide inside a long smooth hollow tube whose inner circular cross-section is just large enough to contain the spheres. The tube has length $2\pi a$ where a is much greater than b, and it is bent in the form of a large circle of radius a and closed on itself. Suppose that the tube is held fixed in a horizontal plane, that S_1 and S_2 are initially touching each other, that S_1 is at rest and that S_2 is projected away from S_1 with speed U. Find, in terms of U, a, and the coefficient of restitution e for collision between the spheres,

(a) the speeds of S_1 and S_2 after the nth collision,

(b) the time that elapses before the nth collision.

Suppose now that the tube is held fixed in a vertical plane and that S_1 and S_2 are initially at its lowest point. S_2 is projected away from S_1 as before. Find the greatest value of U for which S_1 fails to make a complete revolution of the tube after the first collision.

None

Question (1972 STEP IIII Q12)

A circular hoop of mass m is pivoted so as to be able to rotate freely in a horizontal plane about a point O on its circumference. A small, smooth ring of mass km is free to slide on the hoop. Initially the ring lies at P, the opposite end of the diameter through O, and the system is at rest. It is then set in motion by equal and opposite impulses I applied at P to the ring and the hoop. By using the principle of conservation of angular momentum, or otherwise, show that when the ring reaches O, the hoop has rotated through an angle

$$\frac{1}{2}\pi\{1-(1+2k)^{-\frac{1}{2}}\}.$$

None

Question (1958 STEP II Q208)

A light inextensible string, carrying equal masses m at the two ends, hangs over two smooth pegs A, B at the same level and at distance $2a$ apart. A mass $2m$ is attached at the point C of the string which lies midway between A and B, and the system is then released from rest. In the subsequent motion the angle between AC and the vertical is θ. Find the velocity of the mass $2m$ as a function of θ as long as neither mass m has reached the corresponding peg. Find also the tension in the string when $\theta = \frac{1}{4}\pi$.

None

Question (1965 STEP III Q6)

Two particles of equal mass are joined by a light inextensible string of length $\pi a/3$. Initially they rest in equilibrium with the string across the top of a smooth circular cylinder of radius a. The particles are then slightly disturbed from rest, the string remaining taut. Find the position of the particles when the first one leaves the cylinder.

None

Question (1932 STEP I Q109)

A flat disc, with its plane horizontal, is spinning in frictionless bearings at an angular velocity ω_1 about a vertical axis through its centre, its moment of inertia about that axis being I. A uniform ring of mass m and radius R, with its plane horizontal and its centre on the axis of the disc, is lowered on to the latter while spinning in its own plane about its centre with an angular velocity ω_2 in the opposite direction to ω_1. If the coefficient of friction between the ring and the disc be μ, derive an expression for the time during which relative slipping will continue.

Question (1932 STEP I Q109)

Two particles of masses m and m' are joined by a light inextensible string of length $a+b$ and rest on a smooth horizontal plane at points A, B at distances a, b from a smooth vertical peg O round which the string passes so that initially the two portions OA, OB are at right angles. Shew that if the first particle is projected with velocity u parallel to OB, its distance r from O at time t is given by $\dot{r}^2 = a^2 + \frac{m}{m+m'}u^2t^2$ if the string is still in contact with the peg.

Question (1914 STEP I Q109)

A vertical iron door, 6 feet high, 4 feet broad and 1 inch thick, and weighing 490 pounds per cubic foot, is swinging to, its outer edge moving at 6 feet per second. Neglecting friction, find the least steady force which, applied at its outer edge, will stop it while it swings through 10 degrees.

Question (1915 STEP I Q206)

If A and B are points on a rod which is moving in any way in a plane, and if Oa and Ob represent the velocities of A and B at any instant, prove that ab is perpendicular to AB. If C is any other point on the rod and if c divides ab in the same ratio as that in which C divides AB, prove that Oc represents the velocity of C at the same instant.

PQ, QR, RS are three rods in a plane jointed together at Q and R, and with the ends P and S jointed to fixed supports. If a triangle Oqr is drawn with Oq, qr, ro perpendicular to PQ, QR, RS respectively for any position of the rods, prove that as the rods move through this position Oq and Or represent on the same scale the velocities of Q and R.

Question (1918 STEP I Q210)

Two particles A, B, whose masses are m_1, m_2, are tied to the ends of an elastic string whose natural length is a, and they are placed on a smooth table so that $AB = a$. If B is now projected with velocity v in the direction AB, prove that the string will become slack after a time

$$\pi\sqrt{\frac{m_1 m_2 a}{(m_1+m_2)\lambda}},$$

and that the maximum value of the tension of the string is equal to

$$v\sqrt{\frac{m_1 m_2 \lambda}{(m_1+m_2)a}},$$

λ being the modulus of elasticity of the string.

Question (1921 STEP I Q208)

A flywheel of mass M is made of a solid circular disc of radius a. Find its kinetic energy when it rotates n times a second. A ring of radius b is mounted on a shaft in line with the axis of the flywheel, and is driven by an engine at n' revolutions a second. It can be pressed against the flywheel so as to act as a clutch. If the pressure is P and the coefficient of friction μ, find how long it takes for the flywheel to get up full speed from rest, and find the rate at which the engine does work during the process.

Question (1923 STEP I Q207)

Two flywheels, whose radii of gyration are in the ratio of their radii, are free to revolve in the same plane, a belt passing round both. Initially one, of mass m_1 and radius a_1, is rotating with angular velocity Ω, and the other, of mass m_2 and radius a_2, is at rest. Suddenly the belt is tightened, so that there is no more slipping at either wheel. Show that the second wheel begins to revolve with angular velocity

$$\frac{m_1 a_1}{(m_1+m_2)a_2}\Omega.$$

Question (1941 STEP I Q210)

A rigid body is capable of rotation about a fixed axis. Prove that the rate of change of moment of momentum about this axis is equal to the moment of the applied forces about this axis. Point out clearly at which stage of the proof the assumption that the body is rigid is introduced. Two gear wheels, of radii a_1, a_2 and of moments of inertia I_1, I_2, rotate about parallel axes. At an instant when their respective angular velocities are Ω_1, Ω_2 in the same sense the wheels are suddenly put into mesh, with their axes held fixed. Find their new angular velocities.

Question (1935 STEP I Q309)

A particle of mass m is freely suspended by a light rigid wire of length l from a support of mass m which can move freely on a smooth horizontal rail. The system is started by a blow B parallel to the direction of the rail given to the particle. Shew that provided $B < 2m\sqrt{gl}$, the particle will not rise above a certain level below the rail. Shew also that when the inclination of the string to the vertical is θ, the velocity v of the support is given by

$$v = \frac{B}{2m} \pm \frac{1}{2}\cos\theta\sqrt{\frac{B^2}{m^2} - 4gl(1-\cos\theta)2 - \cos^2\theta},$$

and explain the ambiguity of sign.

Question (1934 STEP III Q305)

A rigid body consisting of two equal masses joined by a weightless rod rests on a smooth horizontal table. One of the masses receives a horizontal blow perpendicular to the rod. Prove that each mass describes a cycloid.

If the body is thrown up in the air in any manner, and air resistance is neglected, describe the motion in general terms.

Question (1942 STEP III Q308)

A rigid light rod ABC has three particles of the same mass m attached to it at A, B, C, where $AB = a$ and $BC = b$ $(a > b)$. The rod is moving at right angles to its length with velocity u, when its middle point O is suddenly fixed. Find the impulse at O and prove that there is a loss of energy

$$4mu^2(a^2 + ab + b^2)/(3a^2 + 2ab + 3b^2).$$

Question (1940 STEP I Q410)

Find the moment of inertia of a uniform rectangular lamina about a diagonal in terms of the mass and the lengths of the sides.

A uniform rectangular lamina ABCD of mass M is free to rotate about the diagonal BD, which is horizontal, and a particle of mass m is attached to the lamina at C. When the system is in stable equilibrium an impulse is applied at its mass centre, and perpendicular to the lamina. If the lamina is instantaneously at rest when horizontal, determine the magnitude of the impulse.

Question (1921 STEP II Q704)

A smooth non-circular disc is rotating with angular velocity ω on a smooth horizontal plane about its centre of mass, when it strikes a smooth uniform rod of mass m at the middle point of the rod. Prove that the new angular velocity is

$$\frac{(M+m)I - ep^2Mm}{(M+m)I + p^2Mm}\omega,$$

where M and I are the mass and moment of inertia of the disc, p the perpendicular from its centre of mass to the normal at the point of contact, and e the coefficient of restitution.

Question (1918 STEP III Q704)

A square plate of side a and mass M is hinged about its highest edge, which is horizontal. When at rest it is struck horizontally, at a depth h below the hinge, by a particle of mass m travelling with velocity v. The particle becomes embedded in the plate close to the surface. Determine the subsequent motion of the plate.