

Question (1975 STEP I Q15)

The section of the curve $y = \cosh x$ between $x = 0$ and $x = a$ is rotated about the x -axis. Prove that the numerical value of the curved surface area thus obtained is twice that of the volume enclosed. The curve is now rotated about the y -axis. Calculate the ratio of the numerical values of volume to curved surface area, and show that in this case it depends on a .

Question (1983 STEP I Q1)

Show that

$$\cosh x - \cosh y = 2 \sinh \left(\frac{x+y}{2} \right) \sinh \left(\frac{x-y}{2} \right)$$

Show that the inverse hyperbolic function

$$y = \sinh^{-1} x$$

satisfies the differential equation

$$(x^2 + 1) \frac{d^2 y}{dx^2} + x \frac{dy}{dx} = 0.$$

Question (1969 STEP II Q14)

A mapping of the (X, Y) plane onto the (x, y) plane is given by

$$x = \sin X \cosh Y,$$

$$y = \cos X \sinh Y.$$

Find and sketch the curves in the (x, y) plane which correspond under this mapping to the lines $X = \text{const.}$ and $Y = \text{const.}$ To which curves in the (X, Y) plane do the lines $x = 0, y = 0$ and $x = y$ correspond?

Question (1954 STEP III Q103)

Define the function e^y , and deduce from your definition that, for all values of n , $y^n e^{-y} \rightarrow 0$ as $y \rightarrow \infty$. Examine the behaviour of the following functions as x varies through real values, and in particular discuss their gradients for small positive and negative values of x . Illustrate your results by sketch-graphs.

$$(i) \tanh \frac{1}{x}, \quad (ii) x \tanh \frac{1}{x}.$$

Question (1956 STEP III Q305)

If $u_0 = \sinh \alpha$, $u_1 = \sinh(\alpha + \beta)$ and $u_{n+2} - 2u_{n+1} \cosh \beta + u_n = 0$ for all $n \geq 0$, prove that $u_n = \sinh(\alpha + n\beta)$. Sum the series

$$\sum_{r=0}^n \sinh(\alpha + r\beta)$$

for all values of β .

Question (1914 STEP I Q104)

Give definitions of, and proofs of the simplest properties of, the hyperbolic functions $\cosh x$, $\sinh x$, $\tanh x$. Draw the graphs of the functions and of the inverse functions; and express the inverse functions in terms of logarithms. Explain the parallelism between formulae involving the hyperbolic functions and the corresponding formulae involving the trigonometrical functions $\cos x$, $\sin x$, $\tan x$.

Question (1926 STEP I Q409)

If x is an acute angle and if $y = \log \tan \left(\frac{\pi}{4} + \frac{x}{2} \right)$, prove that $\cos x \cosh y = 1$, and that

$$y = \sin x - \frac{1}{3} \sin 3x + \frac{1}{5} \sin 5x - \dots$$

Question (1930 STEP I Q609)

Define the hyperbolic functions and establish their most important properties, including the expressions for $\sinh(u+v)$ and $\cosh(u+v)$ in terms of $\sinh u$, $\sinh v$, $\cosh u$, $\cosh v$. Prove that the sum of the series

$$\sinh \theta + \tan \theta \sinh 2\theta + \tan^2 \theta \sinh 3\theta + \dots + \tan^{n-1} \theta \sinh n\theta$$

is

$$\frac{\sinh \theta - \tan^n \theta \sinh(n+1)\theta + \tan^{n+1} \theta \sinh n\theta}{\sec^2 \theta - 2 \cosh \theta \tan \theta}.$$

Question (1923 STEP III Q710)

Prove that, if the circle of curvature at any point P on the cardioid $r = a(1 + \cos \theta)$, which has its cusp at O and OA for its axis, cuts the curve again in Q , then

$$\frac{1}{OQ} - \frac{9}{OP} = -\frac{8}{OA}.$$